Maritime Air Defence Firing Tactics

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***Abstract*—A typical firing doctrine is the Shoot-Look-Shoot tactic. In this tactic, the defence launches a salvo of interceptors against the targets (Shoot), assesses the outcomes of the engagements (Shoot-Look), and launches another salvo (Shoot-Look-Shoot) if time and the inventory of interceptors permit. In the open literature, it is often assumed that the targets are identical. This is not always true as targets come in with different ranges, speeds, sizes, cross sections etc. In this paper, we consider two types of targets. Each type of target has a different number engagement opportunities due to their ranges and speeds. Through the use of dynamic programming, a genetic algorithm, and a recursive generating function, we determine the probability of raid annihilation (the probability of neutralizing all of the targets) for two different Shoot-Look-Shoot (SLS) tactics. The first SLS tactic is based on variable size salvos and maximizes the probability of raid annihilation (PRA) for heterogeneous targets. The second SLS tactic is based on fixed-size salvos and is robust as it is independent of the number and types of targets. Theoretical results are validated through some computer simulations.**

NOMENCLATURE

DP	Dynamic Programming
GA	Genetic Algorithm
GF	Generating Function
PRA	Probability of Raid Annihilation
SLS	Shoot–Look–Shoot
SSPK	Single Shot Probability of Kill

I. INTRODUCTION

A. Motivation

Firing tactics are of paramount importance in air defence applications as they can affect the effectiveness of the defence significantly. The choice of the best tactic depends on many factors which often boil down to the single shot probability of kill (SSPK), the number of engagement opportunities, and the inventory of interceptors. In some cases, the defence encounters different types of targets (*i.e.*, heterogeneous in the context of this paper) with different numbers of engagement opportunities associated to each type. One of the critical measures of effectiveness is the probability of raid annihilation (PRA), *i.e.*, the probability of successfully neutralizing all targets. It is required for the defence to rapidly decide on the allocation of interceptors in an optimal manner such that the PRA is maximized. However, the computation of the optimal PRA is nontrivial when the targets are heterogeneous and the number of available interceptors is large. This clearly motivates practical alternatives (as opposed to standard recursive procedures) in determining the effectiveness of a defence for neutralizing heterogeneous targets with a large number of available interceptors.

B. Literature Review

The SLS tactics for minimizing the expected damage to a terminal defence have been pioneered by Soland in 1987 [1], where the limited inventory of interceptors and the number of engagement opportunities are taken into consideration. Several SLS tactics and their extensions are discussed in [2], [3]. Nguyen *et al* in [4] proposed the idea of generating functions to determine the PRA in a SLS tactic with fixed-size salvos. A few extensions on the optimal missile allocation problem are introduced in [5] and in some references therein.

To the best of our knowledge, relatively little work has been conducted for the case when the defence encounters heterogeneous targets. In this work we address this problem by incorporating evolutionary algorithms with a dynamic programming approach to determine the PRA in maritime air defence.

C. Contribution Statement

This paper contributes to the development of two different SLS tactics. First, we propose an extension of Soland's (variable size) salvo tactic for heterogeneous targets. The PRA for heterogeneous targets is maximized using dynamic programming (DP). This idea is then extended to compute the maximum PRA using genetic algorithm (GA) optimization. Second, when the defence employs a number of fixed-size salvos of interceptors, we propose a layer defence SLS tactic for neutralizing both homogeneous and heterogeneous targets. While the former tactic yields an optimal PRA for heterogeneous targets, the latter is a robust tactic which is independent of the number of targets and their types.

D. Organization

The outline of this paper is as follows. In section II, we give some preliminaries and assumptions made throughout the paper. In section III, we describe the variable size salvo SLS tactic and the use of dynamic programming to maximize PRA. In section IV, we show how to implement the genetic algorithm to maximize the PRA for the same tactic as the one in section III. In section V, we model the outcomes of the fixed-size salvo tactic using a generating function. In section VI, we present numerical results. Finally, we conclude in section VII.

II. Preliminaries and Assumptions

We define the SLS firing tactic in maritime air defence as follows. The defence has N interceptors and E engagement opportunities in total to neutralize Q targets. The SSPK of a defending interceptor is denoted by $h \in [0,1]$ and $m = 1-h$ is the "single shot probability of miss". It is assumed that the defence has the perfect kill assessment capability after each shot (salvo of interceptor(s)) fired at a target. This means that the defence has perfect knowledge of the outcome of an engagement. Furthermore, we assume that the defence engages targets in reverse chronological order, with $e,\ e = 0,\ldots,E$, engagement opportunities remaining for the defence to engage targets before the attack takes place [1]. Note that $e = 0$ signifies that there is no more engagement opportunity left for the defence to engage targets. In fact, the SLS strategy ends when all the interceptors are fired, or no more engagement opportunities are left (or no more time left), or all Q targets are neutralized. For positive integers $p,\ q$, with $p \geq q$, we use the standard notation

$$\binom{p}{q} = \frac{p!}{q!(p-q)!}$$

to represent binomial coefficients. Similar to [6] we let

$$\mathrm{bin}(q,m)(i) = \binom{q}{i} m^i h^{q-i},\ i = 0,\ldots,q,$$

to denote the probability of missing i(out of q) targets with i independent trials of interceptors. $\mathbb{N} = \{1,2,\ldots\}$ denotes the set of natural numbers and $\mathbb{N}_0 = \{0\} \cup \mathbb{N}$. The operator $\lfloor\cdot\rfloor(\lceil\cdot\rceil)$ determines the largest (smallest) integer smaller (greater) than or equal to $(\cdot])$.

III. Variable Size Salvo SLS Tactic

In this section, we determine a critical effectiveness, *i.e.*, the probability of raid annihilation (PRA), of a defence when the targets are heterogeneous. In this context, heterogeneous targets are meant to be described by the different types (or groups) of targets based on their kinematics. We define $S(n,q,e)$ as the PRA when there are n interceptors left, q targets survive to attack the defence, and e engagement opportunities remain. The defence launches the optimal number of interceptors in each of the remaining engagement opportunities, for $n = 0,\ldots,N,\ q = 0,\ldots,Q$, and $e = 0,\ldots,E$. $\Pr(j|i,q,e)$ is the probability that j of the q targets survive the eth engagement from the end, $i \leq n$ interceptors are launched in an optimal manner at the eth engagement [1].

Intuitively, $I = \frac{i}{q}$ interceptors are fired at each target. In other words, each of the q targets is fired with a salvo of size I interceptors, which is variable in this tactic. If I is not an integer, which is the case in general, $\underline{q} = q + q\underline{I} - i$ of the q targets are assigned $\underline{I} = \lfloor I \rfloor$ interceptors each and the remaining $\bar{q} = i - q\underline{I}$ are assigned $\bar{I} = \lceil I \rceil$ interceptors each. If the Q targets are homogeneous (*i.e.*, all targets follow the same kinematic model), then the probability that j targets survive the eth engagement is given by

$$\Pr(j|i,q,e) = \begin{cases} \mathrm{bin}(q,m^I)(j), & \text{if } I \in \mathbb{N}_0, \\ \displaystyle\sum_{p=\mathrm{pmin}}^{\mathrm{pmax}} \left[\mathrm{bin}(\underline{q},m^{\underline{I}})(p)\right] \left[\mathrm{bin}(\bar{q},m^{\bar{I}})(j-p)\right], \end{cases} \tag{1}$$

if $I \notin \mathbb{N}_0$, $\mathrm{pmin} = j - \min(j,\bar{q})$, and $\mathrm{pmax} = \min(j,\bar{q})$. Let $S^*(n,q,e)$ be the optimal PRA, for $n = 0,\ldots,N$, $q = 0,\ldots,Q$, and $e = 0,\ldots,E$. Following [1], we employ dynamic programming to optimize the PRA using the recursive relation

$$S(n,q,e) = \max_i \left\{ \sum_{j=0}^{q} \Pr(j|i,q,e) S(n-i,j,e-1) \right\}, \tag{2}$$

for $i = 0,\ldots,n$, with boundary conditions

$$S(n,q=0,e) = 1 \text{ and} \tag{3}$$

$$S(n,q,e=0) = \delta_{q,0}. \tag{4}$$

The interpretation of the boundary condition (3) stems from the fact that the PRA is always one (hundred percent) if the number of targets is zero. However, the boundary condition (4) implies that if the number of targets that survive is non-zero (*i.e.*, $q \neq 0$) and no more engagement opportunities are left ($e = 0$), then it is certain that the defence will be unsuccessful, *i.e.*, the PRA $S(n,q,e=0) = 0$. For the case when $q = 0$ and $e = 0$, the PRA $S(n,q,e=0) = 1$. The Kronecker delta function, $\delta_{q,0}$, in (4) is defined as

$$\delta_{q,0} = \begin{cases} 1, & \text{if } q = 0, \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

The extension of Soland's variable size salvo tactic for maximizing the PRA modeled by (2) is possible when the targets are heterogeneous. For that, we consider two different types of targets: the first type is a group of targets with "fast" speeds and the second type is that with "slow" speeds. The number of targets with "fast" ("slow") speeds is denoted by $q_H(q_L)$. Let $\vec{q}$ denotes the vector of two types of targets, *i.e.*, $\vec{q} = [q_F, q_L]^T$. Clearly, the defence may have two different engagement opportunities for neutralizing these two types of targets. Let us denote them as the vector $\vec{e} = [e_1, e_2]^T$, where $e_1(e_2)$ engagements are possible before the attack by the fast (slow) speed targets and $e_1 \leq e_2$. In this case, the globally optimal PRA is determined using dynamic programming by the following recursive relation:

$$S_H(n,\vec{q},\vec{e}) = \max_{i_1,i_2} \left\{ \sum_{j_1,j_2=0}^{q_F,q_L} \Pr(j_1|i_1,q_F,e_1) \Pr(j_2|i_2,q_L,e_2) S_H(n',\vec{j},\vec{e}-\vec{1}) \right\}, \tag{6}$$

for $i_1 = 0,\ldots,n$, $i_2 = 0,\ldots,n$, $i_1+i_2 \leq n$, $n' = n-(i_1+i_2)$, and $\vec{j} = [j_1,j_2]^T$. The boundary condition for the recursive model (6) is given by

$$S_H(n, \vec{q}, [1, e_2]^T) = \max_{i_1} \{P_{\text{salvo}}(q_F, i_1) S(i_2, q_L, e_2)\},$$

where $P_{\text{salvo}}(q_F, i_1)$ is the probability of neutralizing q_F targets with i_1 interceptors using the Salvo tactic and is given by

$$P_{\text{salvo}}(q_F, i_1) = \left(1 - m^{\bar{I}}\right)^{\bar{q}_1} \left(1 - m^{\underline{I}}\right)^{\underline{q}_1},$$

with $\bar{I} = \lceil i_1/q_F \rceil$, $\underline{I} = \lfloor i_1/q_F \rfloor$, $\underline{q}_1 = q_F + q_F \underline{I} - i_1$ and $\bar{q}_1 = i_1 - q_F \underline{I}$.

It is important to point out that the dynamic programming approach, by construction, given by the model (6) provides a globally optimal PRA for heterogeneous targets. This is because the decision variables (number of interceptors assigned for each type of targets), i_1 and i_2, are chosen by an exhaustive search from $0, \ldots, n$ and the allocation of interceptors to all missing targets at each engagement opportunity through $\Pr(\cdot)$ in model (6) is optimal.

IV. Genetic Algorithm

Clearly, the computation of the PRA using (6) in the previous section suffers from an overwhelming degree of computational complexity when n is large. Hence, a practical alternative solution to deal with such an optimization problem is to employ the Genetic Algorithm optimization technique.

The Genetic Algorithm, pioneered by Holland [7], has been the subject of extensive research due to its global optimization capability based on heuristic search, which mimics the mechanism of the biological evolution by natural selection. Even though the genetic algorithm does not always guarantee global optimal solutions, it is a powerful tool for solving complex problems in many applications [8], [9], [10]. A simple yet detailed implementation of the genetic algorithm for optimizing a general class of objective functions can be found in [11]. Here we present the pseudo code (see Algorithm 1) for maximizing the PRA in (6) using the genetic algorithm with dynamic programming. The key steps in determining the maximum PRA for a fixed single shot miss probability is provided in the function GA in Algorithm 1, where one-point crossover and uniformly distributed mutation operators are used to generate offsprings from a fixed sized populations. The technical details of GA can be sought in [11] and some references therein but omitted here for the conciseness purpose.

A special case of the SLS tactic given by (6) is when the defence launches fixed-size salvos of interceptors at each engagement opportunity. In the following, we detail a layer defence SLS tactic which takes into account the fixed number of salvos of interceptors available to the defence in computing the PRA for homogeneous and heterogeneous targets.

Algorithm 1: Pseudo code to compute PRA using genetic algorithm.

```
Input: n, q⃗ (2-D vector), e⃗ (2-D vector), and single shot
       miss probability m = 1 − h
Output: Maximum PRA, S*(n, q⃗, e⃗, m).
begin
    n̄ :  minimum number of interceptors to execute GA
    if n > n̄ then
        • S*(n, q⃗, e⃗, m) = GA(n, q⃗, e⃗, m)
    else
        • Use dynamic programming (6) to compute
        S*(n, q⃗, e⃗, m).
    return S*(n, q⃗, e⃗, m)

    Function GA (n, q⃗, e⃗, m)
    GEN: total number of generations (stopping criteria)
    p_m : mutation rate 0 < p_m < 1, pop: population array
    • Generate random population of r individuals
    (i_1, i_2)
    • PRA(r); //Compute r PRA values using (6)
    // Select maximum PRA and its corresponding index
    • [PRAmax,pop_index] = max(PRA)
    • sol = pop(pop_index); //select best individual
    repeat
        • Generate offsprings (from each pair of
        population) with a crossover probability to
        generate r_new > r populations
        • Apply mutation operator on each of the new
        population with fixed p_m
        • PRA_new(r_new); //r_new PRA values using (6)
        • [PRAmax_new, pop_index_new] =
        max(PRA_new)
        if PRAmax<PRAmax_new then
            • PRAmax = PRAmax_new
            • pop_index = pop_index_new
            • sol_new =
            pop_new(pop_index);// individual with the
            fastest fitness value
        • Select the best r out of r_new population with
        greater PRA (fitness) values
        • Replace lowest PRA individual with sol_new
    until GEN is reached or satisfactory PRA is found
    return Maximum PRA value and its individual
```

V. Fixed-sized Salvo SLS Tactic

In this section, we determine the PRA when the defence launches fixed-size salvos of interceptors against targets for all of the engagement opportunities. An efficient way to do that is to express the probability distribution, $\Pr(0 \leq X \leq q)$ in terms of a Generating Function (GF) [4], where X is the discrete random variable representing the number of targets to be neutralized. Nguyen *et al* in [4] exploited GFs for solving problems in missile defence. Similar to the variable size salvo tactic presented in section III, we are interested in developing a fixed-size salvo SLS tactic with an inventory of salvos of interceptors, $n_s \geq 0$, available to the defence for a number of engagement opportunities.

For that, we consider a salvo of interceptors of size s such that $sn_s = N$, where N is the total number of interceptors available to the defence. At each engagement opportunity, a target is engaged with one salvo of interceptors yielding two outcomes: the target is neutralized by the salvo with the probability $h_s = 1-(1-h)^s$ or it is missed by the salvo with the probability $m_s = 1 - h_s$.

In the following sections, we determine the PRA for both homogeneous and heterogeneous targets.

A. PRA for Homogeneous Targets

In this case, the defence has the same number of engagement opportunities, E, for all targets. For $E \geq 1$, we define the recursive operator for the e, $1 \leq e \leq E$, remaining engagement opportunities as

$$g_e = \begin{cases} 1 & \text{if } e = 0, \\ g_{e-1} m_s + h_s & \text{if } e > 0. \end{cases} \tag{7}$$

Even though $g_e = 1$, $\forall e > 0$, in (7) given the fact that $m_s + h_s = 1$, it is the recursive structure of g_e that is important to determine the PRA which will be clearer later in this section. The probability distribution for neutralizing q homogeneous targets, $\Pr(0 \leq X \leq q)$, with E remaining engagement opportunities can be expressed in terms of the recursive GF defined by

$$G_E(n_s, q) := \begin{cases} 1 & \text{if } E = 0, \\ (g_E)^{b_0} = (g_{E-1} m_s + h_s)^{b_0} & \text{if } E > 0, \end{cases} \tag{8}$$

where $b_0 = \min(q, n_s)$ and the recursive operator $g_{(\cdot)}$ is defined in (7). If $\Pr(X = \jmath)$ denotes the probability of neutralizing $\jmath$ targets, then the model (8) implies that $G_E(n_s, q) = \Pr(0 \leq X \leq q) = \sum_{\jmath=0}^{q} \Pr(X = \jmath) = 1$. The closed form expression of the recursive GF (8) is presented in Theorem 1 subject to the fact that the defence has n_s salvos of interceptors.

Theorem 1 (Recursive GF): Suppose that the defence has fixed n_s salvos of interceptors to neutralize q targets in maximum E engagement opportunities. Given the recursive operator defined in (7), the closed form expression of the recursive GF (8) for q targets with E engagement opportunities, $(E > 0)$, is given by

$$G_E(n_s, q) = \prod_{e=1}^{E} \sum_{l_e=0}^{b_{e-1}} \operatorname{bin}(b_{e-1}, m_s)(l_e), \tag{9}$$

where $b_{e-1} = \min(l_{e-1}, n_s - \sum_{\kappa=0}^{e-2} b_\kappa)$, for $e = 2, \ldots, E$.

Proof: We prove this Theorem recursively on E. For that, consider the case when the defence has only one engagement opportunity $(E = 1)$ to neutralize targets. Using (8),

$$G_1(n_s, q) = (m_s + h_s)^{b_0},$$

with $b_0 = \min(n_s, q)$ and the corresponding PRA is given by

$$S_1(n_s, q) = \begin{cases} 1 & \text{if } q = 0, \\ 0 & \text{if } 0 \leq n_s < q, \\ h_s^q & \text{if } n_s \geq q. \end{cases} \tag{10}$$

Now consider the case when $E > 1$, $n_s > q$, and the defence engages the targets in reverse chronological order. The defence survives the first engagement opportunity (from the end) with its available interceptors and has more interceptors for a possible second engagement of the targets that are missed in the first engagement. Following (8) and (7), the GF at the second engagement opportunity is given by

$$G_2(n_s, q) = \sum_{l_1=0}^{q} (m_s + h_s)^{b_1} \operatorname{bin}(q, m_s)(l_1), \tag{11}$$

where $b_1 = \min(l_1, n_s - q)$. The GF in the second engagement opportunity (11) implies that l_1 targets are missed in the first engagement opportunity but a maximum of b_1 missed targets can be re-engaged with the remaining salvos of interceptors. Suppose that the defence still has some salvos of interceptors to re-engage targets that are missed during the second engagement opportunity. Hence, the GF for the third engagement opportunity is expressed as

$$G_3(n_s, q) = \sum_{l_1=0}^{q} \operatorname{bin}(q, m_s)(l_1) \sum_{l_2=0}^{b_1} (m_s + h_s)^{b_2} \operatorname{bin}(l_1, m_s)(l_2), \tag{12}$$

where $b_2 = \min(l_2, n_s - q - b_1)$. Similar to model (11), the GF (12) implies that l_2 targets are missed in the second engagement opportunity, however, a maximum of b_2 targets will be re-engaged by the defence as it is limited by the inventory of interceptors. If this SLS process continues and $b_1, b_2, \ldots, b_{E-2} > 0$, then the defence survives the $(E-1)$th engagement opportunity with its available salvos of interceptors. Consequently, the GF for the Eth engagement opportunity is given by the following expression:

$$\begin{aligned} G_E(n_s, q) = &\sum_{l_1=0}^{q} \operatorname{bin}(q, m_s)(l_1) \sum_{l_2=0}^{b_1} \operatorname{bin}(b_1, m_s)(l_2) \times \\ &\sum_{l_3=0}^{b_2} \operatorname{bin}(b_2, m_s)(l_3) \sum_{l_4=0}^{b_3} \operatorname{bin}(b_3, m_s)(l_4) \times \\ &\vdots \\ &\sum_{l_{E-1}=0}^{b_{E-2}} (m_s + h_s)^{b_{E-1}} \operatorname{bin}(b_{E-2}, m_s)(l_{E-1}), \end{aligned}$$

where $b_{E-1} = \min(l_{E-1}, n_s - q - \sum_{\kappa=1}^{E-2} b_\kappa)$. The proof is completed by the fact that

$$(m_s + h_s)^{b_{E-1}} = \sum_{l_E=0}^{b_{E-1}} \operatorname{bin}(b_{E-1}, m_s)(l_E).$$

and $b_0 = q < n_s$.

■

Note that if $b_{e-1} = l_{e-1}$, for $e = 2, \ldots, E$, then the defence has no deficit of interceptors and is able to re-engage all the missing targets until the last, Eth, engagement opportunity. In order to determine the PRA, we expand the GF of the eth,

$1 \le e \le E$, engagement opportunity and sum all the terms that involve h_s^q, *i.e.*, $\sum_{\imath,\jmath} c_{\imath\jmath} m_s^\imath h_s^\jmath$, such that $\jmath = q$, $\imath \le n_s$, and $c_{\imath\jmath}$ is the coefficient. Hence, the PRA at the eth, $1 \le e \le E$, engagement opportunity is determined by

$$S_e(n_s, q) = G_e(n_s, q)\Big|_q = \sum_{\imath,\jmath} c_{\imath\jmath} m_s^\imath h_s^\jmath\Big|_{\jmath=q}. \quad (13)$$

We illustrate the above recursive computation of the PRA using the following Example.

Example 1 (PRA using fixed-size salvo SLS tactic): Consider the case when the defence has $n_s = 6$ salvos of interceptors with the salvo size of $s = 2$, hence, the total of 12 interceptors. The defence is required to neutralize $q = 3$ homogeneous targets with the maximum of $E = 3$ engagement opportunities. We will determine the expression for the PRA, $S_{E=3}(n_2 = 6, q = 3)$, using the recursive GF (9) illustrated above.

Obviously, for the first engagement the GF is

$$G_1(6,3) = (m_2 + h_2)^3 = m_2^3 + 3m_2^2 h_2 + 3m_2 h_2^2 + h_2^3. \quad (14)$$

From Eq. (14), the expression for the PRA, using the generalized model (13), is simply

$$S_1(6,3) = h_2^3. \quad (15)$$

The first, second, and the third terms of Eq. (14) respectively mean that the defence has missed three, two, and one, targets. Hence, each of the missing targets can be re-engaged with the remaining three salvos of interceptors.

Using (11), we write the GF for the second engagement opportunity as

$$G_2(6,3) = h_2^3 + 3(m_2 + h_2)m_2 h_2^2 + 3(m_2 + h_2)^2 m_2^2 h_2 + (m_2 + h_2)^3 m_2^3, \quad (16)$$

and the PRA using (13),

$$S_2(6,3) = h_2^3(1 + m_2)^3. \quad (17)$$

The second, third, and fourth term of Eq. (16), respectively, reveal that the missing target(s) have been re-engaged for the second time. In case the second engagement misses the target(s), the defence employs its third engagement opportunity, which gives the GF (using (12)) as

$$G_3(6,3) = h_2^3 + 3m_2 h_2^2\left[h_2 + (m_2 + h_2)m_2\right] + 3m_2^2 h_2\left[h_2^2 + 2(m_2 + h_2)m_2 h_2 + (m_2 + h_2)m_2^2\right] + (m_2 + h_2)^3 m_2^3. \quad (18)$$

In fact, the GF expression in (18) is the direct consequence of the generalized recursive GF (9) for $n_s = 6,\ q = 3$, and $E = 3$. Using the model (13), the corresponding PRA is given by

$$S_3(6,3) = h_2^3(1 + 3m_2 + 6m_2^2 + 7m_2^3). \quad (19)$$

From Eqs. (15), (17) and (19), we observe that $S_3(6,3) \ge S_2(6,3) \ge S_1(6,3)$, $\forall h \in [0,1]$. This is true as the PRA increases with the number of engagement opportunities.

In the next example (see Example 2), we show that even though the defence has sufficient engagement opportunities, it can not always exploit all the engagement opportunities due to the deficit of interceptors for neutralizing targets.

Example 2: Consider $n_s = 5$, $q = 3$, and $E = 3$. The direct consequence of the recursive GF (9) in Theorem 1 gives

$$G_3(5,3) = h_2^3 + 3m_2 h_2^2\left[h_2 + m_2(m_2 + h_2)\right] + 3m_2^2 h_2 (m_2 + h_2)^2 + (m_2 + h_2)^2 m_2^3. \quad (20)$$

The PRA (using Eq. (13)) in this case is

$$S_3(5,3) = h_2^3(1 + 3m_2 + 6m_2^2).$$

Both examples (Example 1 and 2) show the power of the recursive GF (9) in determining the PRA using the generalized expression (13).

B. PRA for Heterogeneous Targets

As in the case of variable size salvo SLS tactic illustrated in section III, each type of target has a different number of engagement opportunities. As before, we consider only two types of targets (“fast” and “slow”) with the number of engagement opportunities E_1 and E_2, $(E_1 \le E_2)$ respectively.

In order to determine the PRA for heterogeneous targets, we define the recursive GF as follows:

$$G_{E_1,E_2}(n_s, q_F, q_L) := G_{E_1}(n_s, q_F) G_{E_2}(n_s, q_L). \quad (21)$$

Similar to the case with homogeneous targets, we assume that the defence has n_s salvos of interceptors available. Following Theorem 1, the closed-form expression of (21) is given in the following Corollary.

Corollary 1: Given the fact that the recursive GF (9) exists for each type of targets (“fast” or “slow” targets). There are $E_1(E_2)$ number of engagement opportunities for the defence to engage “fast” (“slow”) targets and $E_1 \le E_2$. The closed-form expression of the recursive GF (21) for heterogeneous targets is given by

$$\begin{aligned} G_{E_1,E_2}(n_s, q_F, q_L) = \\ &\sum_{l_1^{(1)}=0}^{b_0^{(1)}} \mathrm{bin}(b_0^{(1)}, m_s)(l_1^{(1)}) \sum_{l_1^{(2)}=0}^{b_0^{(2)}} \mathrm{bin}(b_0^{(2)}, m_s)(l_1^{(2)}) \times \\ &\sum_{l_2^{(1)}=0}^{b_1^{(1)}} \mathrm{bin}(b_1^{(1)}, m_s)(l_2^{(1)}) \sum_{l_2^{(2)}=0}^{b_1^{(2)}} \mathrm{bin}(b_1^{(2)}, m_s)(l_2^{(2)}) \times \\ &\vdots \\ &\sum_{l_{E_1}^{(1)}=0}^{b_{E_1-1}^{(1)}} \mathrm{bin}(b_{E_1-1}^{(1)}, m_s)(l_{E_1}^{(1)}) \sum_{l_{E_1}^{(2)}=0}^{b_{E_1-1}^{(2)}} \mathrm{bin}(b_{E_1-1}^{(2)}, m_s)(l_{E_1}^{(2)}) \times \\ &G_{E_2-E_1}\left(n_s - \sum_{\kappa=0}^{E_1-1}(b_\kappa^{(1)} + b_\kappa^{(2)}), l_{E_1}^{(2)}\right), \quad (22) \end{aligned}$$

where $b_0^{(1)} = \min(q_F, n_s)$, $b_0^{(2)} = \min(q_L, n_s - b_0^{(1)})$,

$$b_{e_1}^{(1)} = \min\left(l_{e_1}^{(1)}, n_s - \sum_{\kappa=0}^{e_1-1}(b_\kappa^{(1)} + b_\kappa^{(2)})\right),$$

$$b_{e_1}^{(2)} = \min\left(l_{e_1}^{(2)}, n_s - \sum_{\kappa=0}^{e_1-1}(b_\kappa^{(1)} + b_\kappa^{(2)}) - b_{e_1}^{(1)}\right),$$

for $e_1 = 1, \ldots, E_1 - 1$, and $G_{E_2-E_1}((\cdot), \cdot)$ in (22) is directly derived using (8) and (9). Note the GF (22) clearly gives priority to the "fast" targets as opposed to the "slow" targets. This makes sense since the defence has less number of engagement opportunities for the "fast" targets and needs to neutralize this type of targets first.

The proof of this Corollary follows from the similar technique presented in the proof of Theorem 1 except the fact that we are now considering heterogeneous targets.

The PRA for heterogeneous targets using fixed-size salvos of interceptors is determined as follows. We expand the recursive GF (22) for the e_1th, $1 \le e_1 \le E_1$, and e_2th, $1 \le e_2 \le E_2$, engagement opportunities and sum all the terms that involve $h_s^{q_F+q_L}$, *i.e.*, $\sum_{\imath,\jmath} c_{\imath\jmath} m_s^\imath h_s^\jmath$, such that $\jmath = q_F + q_L$, $\imath \le n_s$, and $c_{\imath\jmath}$ is the coefficient. The expression for the PRA is as follows:

$$S_{e_1,e_2}(n_s, q_F, q_L) = G_{e_1,e_2}(n_s, q_F, q_L)\Big|_{q=q_F+q_L} = \sum_{\imath,\jmath} c_{\imath\jmath} m_s^\imath h_s^\jmath\Big|_{\jmath=q=q_F+q_L}. \quad (23)$$

For convenience, we determine the expression for the recursive GF (22) and the corresponding PRA through Example 3.

Example 3 (Fixed-size salvo SLS tactic (heterogeneous)): Consider a simple scenario with $n_s = 4$ $(s = 2, N = 8)$. The defence aims to neutralize $q_F = 1$ "fast" target and $q_L = 1$ "slow" target with the maximum of $E_1 = 1$ and $E_2 = 2$ engagement opportunities, respectively. We will employ the model (23) to determine the expression for the PRA, $S_{1,2}(n_2 = 4, q_F = 1, q_L = 1)$, using the recursive GF (22).

Note that the direct consequence of Eq. (22) yields the following expression for the GF

$$G_{1,2}(4,1,1) = \sum_{l_1^{(1)}=0}^{1} \mathrm{bin}(1, m_s)(l_1^{(1)}) \times \sum_{l_1^{(2)}=0}^{1} \mathrm{bin}(1, m_s)(l_1^{(2)}) G_1\left(2, l_1^{(2)}\right),$$

where $G_1\left(2, l_1^{(2)}\right)$ is derived from Eq. (9), which gives

$$G_{1,2}(4,1,1) = h_2\left[h_2 + m_2(h_2 + m_2)\right] + m_2\left[h_2 + m_2(h_2 + m_2)\right].$$

Using the generalized PRA model (23) for heterogeneous targets, the expression for the PRA is

$$S_{1,2}(4,1,1) = G_{1,2}(4,1,1)\Big|_{q=2} = h_2^2(1 + m_2).$$

In the following section, we present numerical results for two SLS tactics illustrated above.

VI. Computational Results

The effectiveness of the two SLS tactics developed in sections III and V is validated through the computations of the PRA for two different types of targets. Note that for the second SLS tactic (fixed-size salvo SLS tactic developed in section V), we provide the PRA for both homogeneous and heterogeneous targets.

A. Variable Size Salvo SLS Tactic for Heterogeneous Targets

The performance of Soland's variable size salvo SLS tactic is tested in determining the maximum PRA for neutralizing heterogeneous targets (*i.e.*, "fast" and "slow" targets) using the dynamic programming and genetic algorithm optimization techniques as illustrated in section III. Note that, in the case of the genetic algorithm, the results are based on 30 computer runs.

We assume a total of five incoming targets, where the number of targets with "fast" speed is $q_F = 2$ and with "slow" speed is $q_L = 3$. The parameters of the genetic algorithm given in Algorithm 1 are set as follows: population size $r = 10$, number of generations $GEN = 5$, mutation rate $p_m = 0.6$, and the offset $\bar{n} = 40$.

We assume three possible sets of engagement opportunities, *i.e.*, $[e_1 = 1, e_2 = 2]^T$, $[e_1 = 2, e_2 = 3]^T$, and $[e_1 = 3, e_2 = 4]^T$, satisfying $e_1 \le e_2$. The total number of available interceptors to the defence is $N = 20$. Fig. 1 shows the performance in computing the PRA for neutralizing five targets with three different sets of engagement opportunities using the genetic algorithm presented in Algorithm 1. We considered the SSPK range $h \in [0, 1]$. As can be seen from Fig. 1, it is natural that the PRA with engagement opportunities $e_1 = 1,\ e_2 = 2$ is always less than that with $e_1 = 2,\ e_2 = 3$ since the defence has more engagement opportunities for the second case. For the same reason, the PRA with the engagement opportunities $e_1 = 3$ and $e_2 = 4$ should be higher than that with the engagement opportunities $e_1 = 2$ and $e_2 = 3$ as evidenced by Fig. 1.

As for the computational complexity, Fig. 2 summarizes the performance in computing the PRA with both the genetic algorithm and the dynamic programming optimization for fixed SSPK, $h = 0.3$, $q_F = 2,\ a_2 = 3$, and $e_1 = 2,\ e_2 = 3$ as the number of available interceptors varies. Using an on-figure magnifier in Fig. 2(a) at $n = 25$, we see that the PRA using genetic algorithm optimization is in good agreement with that using dynamic programming optimization (which gives a global optimal PRA). Interestingly, the difference is less than 0.5% which is practically negligible in the sense that the accuracy of the SSPK (*i.e.*, h in our case) does not likely yield an accuracy of 0.5% in the PRA. However, the computation of PRA is significantly faster using genetic algorithm when the total number of available interceptors is large. Fig. 2(b) reveals the time to compute the PRA for different numbers of interceptors using a personal computer with the Intel(R)

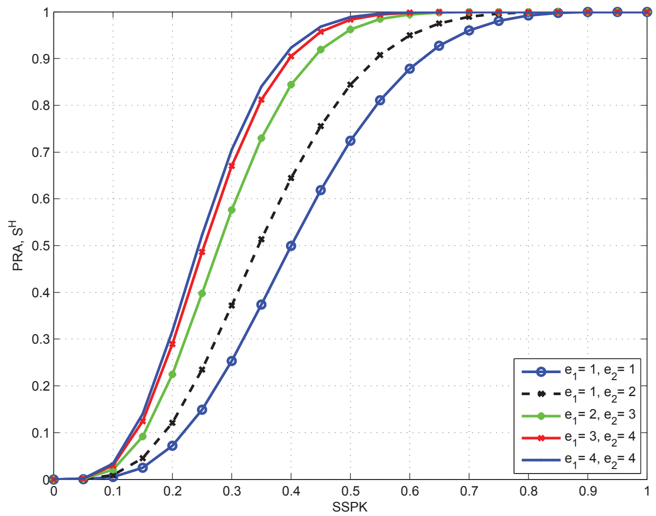
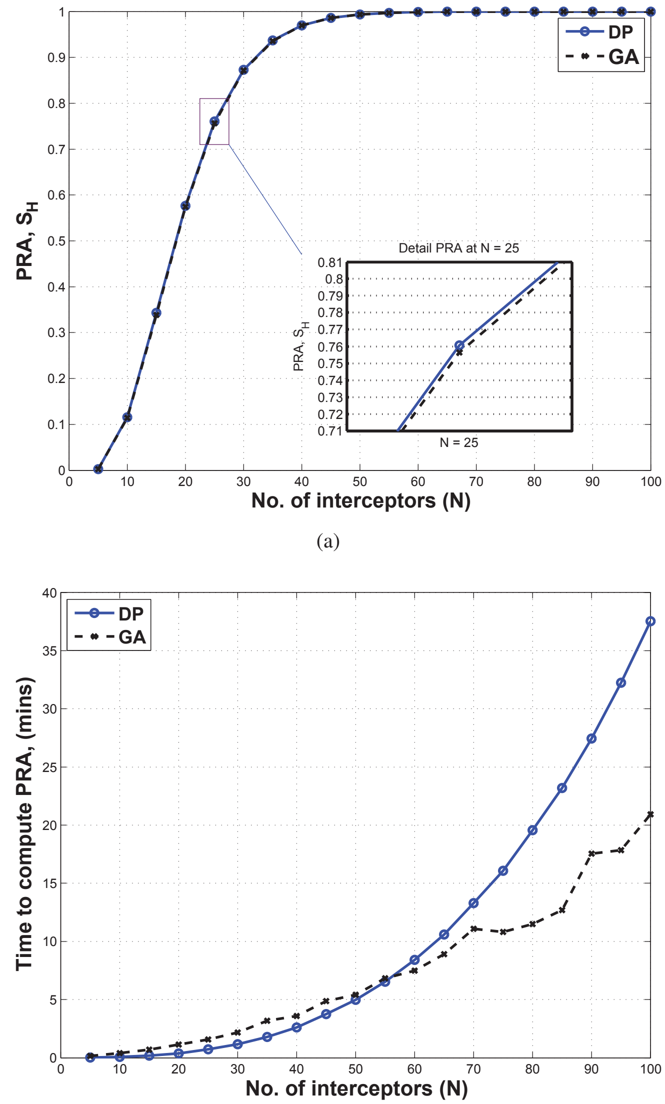


Fig. 1: Performance in computing PRA with genetic algorithm.



(a)

(b)

Fig. 2: Performance in computing PRA with both genetic algorithm and dynamic programming for SSPK $= 0.3$, $q_F = 2$, $q_L = 3$, and $e_1 = 2$, $e_2 = 3$ (a) PRA vs. number of available interceptors, and (b) time to compute PRA vs. number of available interceptors.

Core(TM) i3–2350M CPU2.30 GHz processor running 64-bit Operating System (Windows 7) and 8 GB of installed memory. As can be seen, the time to compute PRA using genetic algorithm is higher than that using dynamic programming for a small number of available interceptors (for $N < 55$). This is simply because Algorithm 1 has to complete the computation for all generations and populations regardless of the value of number of interceptors. When $N = 80$, for instance, the time to compute the PRA is ≈ 20 mins using the dynamic programming optimization and it is less than 12 mins using the genetic algorithm optimization technique. Furthermore, the time complexity is monotonically increasing using the dynamic programming optimization technique as expected, while it is not the case using the genetic algorithm optimization when N is large.

The above results clearly demonstrate that the genetic algorithm is much faster than the dynamic programming optimization for a large number of available interceptors to the defence while maintaining near global optimality. This will be significant in decision making processes as the defence has only a limited time to respond to an air attack. However, this will depend on the computational power of the processors on-board a ship.

B. Performance Fixed-sized Salvo SLS Tactic

1) PRA for Homogeneous Targets: Following the theoretical results presented in section V-A, we consider the case when the defence has $n_s = 10$ salvos of interceptors with the salvo size $s = 2$ (SSPK, $h \in [0, 1]$) and it launches salvos to neutralize $q = 5$ homogeneous targets in maximum $E = 4$ engagement opportunities. Fig. 3 shows the PRAs, $S_1(10, 5)$, $S_2(10, 5)$, $S_3(10, 5)$, and $S_4(10, 5)$ for engagement opportunities, $E = 1$, $E = 2$, $E = 3$, and $E = 4$, respectively. The PRAs, for each engagement opportunity, are computed using the model (13), where the corresponding recursive generating functions are derived directly from the model (9). It is clear from Fig. 3 that the PRA is higher when the defence has more engagement opportunities, as expected. For instance, when SSPK, $h = 0.5$, the PRAs for $E = 1$, $E = 2$, $E = 3$, and $E = 4$ are ≈ 0.23, ≈ 0.72, ≈ 0.92, and ≈ 0.96, respectively.

2) PRA for Heterogeneous Targets: We repeat the same scenario as presented in section VI-A except that we use $n_s = 10$ salvos of interceptors with salvo size $s = 2$ ($N = 20$) to determine the PRA for heterogeneous targets. The results are sumarized in Fig. 4, which immediately follows the generalized PRA model (23). In order to validate the results, we show PRAs, $S_{1,2}(10, 2, 3)$, $S_{2,3}(10, 2, 3)$, and $S_{3,4}(10, 2, 3)$ are bounded by the PRAs $S_{1,1}(10, 2, 3)$, $S_{4,4}(10, 2, 3)$. Note that $S_{1,1}(10, 2, 3)$ and $S_{4,4}(10, 2, 3)$ mimic the PRAs as if all five targets are homogeneous. As expected, PRAs for

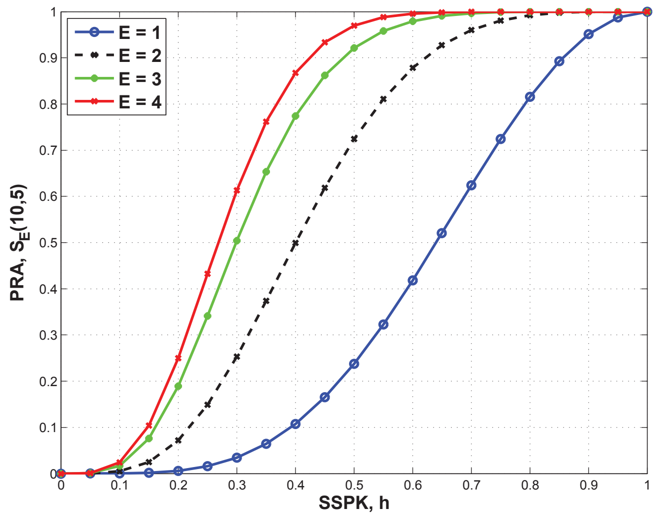


Fig. 3: PRA using fixed-size salvo SLS tactic for homogeneous targets.

the engagement opportunities $(E_1 = 1,\ E_2 = 2)$, $(E_1 = 2,\ E_2 = 3)$, $(E_1 = 3,\ E_2 = 4)$ are strictly non-decreasing for neutralizing two types of targets with $q_F = 2$ and $q_L = 3$.

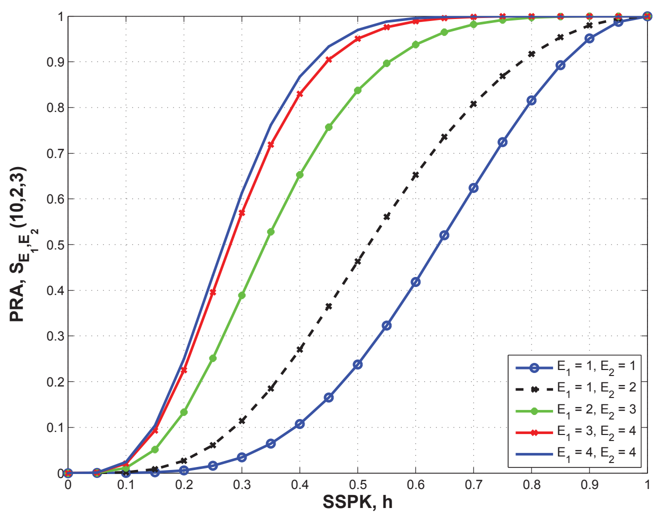


Fig. 4: PRA using fixed-size salvo SLS tactic for heterogeneous targets.

The performance of the fixed-size salvo SLS tactic in determining the PRA clearly demonstrates that this tactic is robust in the sense that it gives the maximum PRA regardless of the number and type of targets.

VII. Conclusion

In this paper, we developed two different SLS tactics which can be applied to maritime air defence. The first SLS tactic is an extension of Soland's tactic where we consider heterogeneous targets. The time complexity in computing the PRA is compared using two different optimization techniques: dynamic programming and genetic algorithm. It is shown that the genetic algorithm optimization outperforms the dynamic programming in terms of computational overhead (only for large numbers of interceptors). This is significant as the defence has only a limited time to make decisions and to engage the targets. Even though we considered two different target kinematics, the expression can be extended to compute the PRA for n different target kinematics.

The second SLS tactic is proposed for robustness when the defence is uncertain about the identity of targets and, hence, it launches fixed-size salvos to targets at each engagement opportunity. We have developed a recursive generating function to determine the PRA and other metrics for this tactic.

A possible future research direction of these two tactics is to incorporate stochastic nature on the number of incoming targets.

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References

[1] R. Soland, "Optimal terminal defence tactics when several sequential engagements are possible," *Operations Research*, vol. 35, no. 4, July 1987.

[2] Y. Aviv and M. Kress, "Evaluating the effectiveness of shoot–look–shoot tactics in the presence of incomplete damage information," *Military Operations Research*, vol. 3, no. 1, Jan 1997.

[3] K. Glazebrook and A. Washburn, "Shoot–look–shoot: A review and extension," *Operations Research*, vol. 52, no. 3, May 2004.

[4] B. Nguyen, P. A. Smith, and D. Nguyen, "An engagement model to optimize defense against a multiple attack assuming perfect kill assessment," *Naval Research Logistics*, vol. 44, no. 7, pp. 687–697, oct. 1997.

[5] O. Karasakal, N. E. Ozdemirel, and L. Kandiller, "Shoot–look–shoot: A review and extension," *Naval Research Logistic*, vol. 58, no. 3, pp. 305–322, June 2011.

[6] S. Bourn, "Probabilistic shoot-look-shoot combat models," Ph.D. dissertation, School of Mathematical Sciences, The University of Adelaide, May 2012.

[7] J. H. Holland, *Adaptation in Natural and Artificial Systems*. Ann Arbor, MI: University of Michigan press, 1975.

[8] H. Roy, A. McGordon, and P. Jennings, "A generalized powertrain design optimization methodology to reduce fuel economy variability in hybrid electric vehicles," *IEEE Transactions on Vehicular Technology*, vol. 63, no. 3, pp. 1055–1070, March 2014.

[9] H. Chaoui and S. Miah, "Maximum power point tracking of wind turbines with neural networks and genetic algorithms," in *40th Annual Conference of IEEE Industrial Electronics Society*, October 2014.

[10] M. Montazeri-Gh, A. Poursamad, and B. Ghalichi, "Application of genetic algorithm for optimization of control strategy in parallel hybrid electric vehicles," *Journal of the Franklin Institute*, vol. 343, no. 1, pp. 420–435, Feb. 2006.

[11] C. Guo and X. Yang, "A programming of genetic algorithm in matlab7.0," *Modern Applied Science*, vol. 5, no. 1, pp. 230–235, Feb. 2011.